Discussion of nodding speed optimization

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Cologne, Dec.14th, 1992
(revised, Jan.20th, 1993)

Summary

The nodding of a telescope is investigated with the goal to minimize the time needed for any given nod amplitude. For this an algorithm is presented, which is particularly useful for telescope drives with acceleration, deceleration and velocity limitations. In order to avoid unnecessary settling time requirements the algorithm is optimized in order to achieve settling already during deceleration under full regulator control. The acceleration is not regulated, but is at full power instead. In case the position information is available in larger time intervals only, the algorithm extrapolates the position information and thereby reduces delay problems. The algorithm is tested by computer simulation and is also in use at the Gornergrat KOSMA telescope. The method is proposed for the control of the SWAS satellite.

Introduction

The control of a closed loop under certain limitations of maximum speed and/or maximum acceleration is a familiar problem to all, who try to move a telescope as fast as possible from one position to the next. The reason is that all motors used to drive the telescope develop a limited maximum torque, since there is a limit to the maximum current through the coils of the motors. In order to move the device with minimum time from position X_1 to X_2 (eventually at speeds V_1 and V_2 respectively), it is obvious that this can be achieved only on a parabolic position curve, which is automatically generated when moving under maximum acceleration A_M and deceleration D_M respectively. In most cases there exists also a maximum speed V_M, which is also a result of the motor properties. (For a spacecraft, controlled with reaction wheels, the same conditions should apply as for a normal telescope drive.) We can write for the position at maximum acceleration:

$$X(t) = X_1 + V_1 \cdot t + \frac{A_M}{2} \cdot t^2$$
$$0 \leq t \leq t_1$$
$$V(t) = V_1 + A_M \cdot t$$

For a movement from position X_1 (with velocity V_1) to position X_2 (with velocity V_2) we can solve now for the time t_1, where the acceleration has to be inverted to maximum deceleration, so that the position X_2 is reached at the best possible time t_2. Deceleration can be described with:

$$X(t) = X_2 - V_2 \cdot (t_2 - t) - \frac{D_M}{2} \cdot (t_2 - t)^2$$
$$t_1 \leq t \leq t_2$$
$$V(t) = V_2 + D_M \cdot (t_2 - t)$$

Velocity and position have to be identical at time t_1 in both, the acceleration and deceleration equations. Combining the two velocity equations at time t_1 we find for the deceleration time interval

$$t_2 - t_1 = \frac{A_M}{D_M} \cdot t_1 + \frac{V_1 - V_2}{D_M}.$$

Together with the position equations, again used at time t_1, one finds now for t_1:

$$t_1 \;=\; \frac{1}{A_M} \cdot \left[\sqrt{\frac{A_M \cdot D_M}{A_M + D_M} \cdot \left\{\frac{V_1^2}{A_m} + \frac{V_2^2}{D_m} + 2 \cdot |X_2 - X_1|\right\}} \;-\; V_1\right]$$

The minimum deceleration time t_2-t_1 needed under the given circumstances is therefore also found:

$$t_2 - t_1 \;=\; \frac{1}{D_M} \cdot \left[\sqrt{\frac{A_M \cdot D_M}{A_M + D_M} \cdot \left\{\frac{V_1^2}{A_m} + \frac{V_2^2}{D_m} + 2 \cdot |X_2 - X_1|\right\}} \;-\; V_2\right]$$

Thus we have for the full time needed:

$$t_2 \;=\; \sqrt{\left[\frac{1}{A_M} + \frac{1}{D_M}\right] \cdot \left\{\frac{V_1^2}{A_m} + \frac{V_2^2}{D_m} + 2 \cdot |X_2 - X_1|\right\}} - \frac{V_1}{A_M} - \frac{V_2}{D_M}$$

For the following we consider only the simpler case of V_1 and V_2 set to zero. The system should now follow the following path:

$$X(t) \;=\; \begin{cases} X_1 & t \;<\; 0 \\ X_1 + \frac{A_M}{2} \cdot t^2 \cdot Sign(X_2 - X_1) & 0 \;\leq\; t \;<\; t_1 \\ X_2 - \frac{D_M}{2} \cdot (t_2 - t)^2 \cdot Sign(X_2 - X_1) & t_1 \;\leq\; t \;<\; t_2 \\ X_2 & t_2 \;\leq\; t \end{cases}$$

The velocity develops like (sawtooth):

$$V(t) \;=\; \begin{cases} 0 & t \;<\; 0 \\ A_M \cdot t \cdot Sign(X_2 - X_2) & 0 \;\leq\; t \;<\; t_1 \\ D_M \cdot (t_2 - t) \cdot Sign(X_2 - X_2) & t_1 \;\leq\; t \;<\; t_2 \\ 0 & t_2 \;\leq\; t \end{cases},$$

and the acceleration is:

$$A(t) \;=\; \begin{cases} 0 & t \;<\; 0 \\ A_M \cdot Sign(X_2 - X_1) & 0 \;\leq\; t \;<\; t_1 \\ -D_M \cdot Sign(X_2 - X_1) & t_1 \;\leq\; t \;<\; t_2 \\ 0 & t_2 \;\leq\; t \end{cases}$$

In case the two positions are rather far apart it is very likely, that a maximum velocity V_M is reached, which is limited by the maximum speed of the motors. This happens, if we have for the nod amplitude:

$$|X_2 - X_1| \;>\; \frac{V_M^2}{2} \cdot \left[\frac{1}{D_M} + \frac{1}{A_M}\right]$$

The total time needed for a nod from X_1 to X_2 is then:

$$t_{tot} \;=\; \frac{|X_2 - X_1|}{V_m} + \frac{V_M}{2} \cdot \left[\frac{1}{D_M} + \frac{1}{A_M}\right]$$

The time intervals Δt_A and Δt_D, during which acceleration or deceleration occur are then:

$$\Delta t_A \;=\; \frac{V_M}{A_M}, \qquad \Delta t_D \;=\; \frac{V_M}{D_M}$$

For a real system it is not very convenient to calculate the best path in advance. Instead, it is much better for the purposes of a servo loop to derive the correct information from the actual position of the telescope or the spacecraft respectively. For this we have to identify the location, from which a maximum deceleration brings the telescope right to the point with the correct velocity.

The algorithm

In general, there is a schedule for the telescope positions, represented by the desired position $S(t)$ (eventually with desired velocity $V_S(t)$), which e.g. defines the Position S_1 for times smaller than zero and the position S_2 for larger t. This is the typical situation for a simple nod between two positions. (Keep in mind that $S(t)$ is the desired position and $X(t)$ the actual position.) We assume that at $t > 0$ the system starts with maximum acceleration A_M. In order to find the precise time t_1, or better, the correct actual position $X(t_1)$, where acceleration stops and the maximum deceleration has to start, we write:

$$S(t_1+T) \;=\; X(t_1+T) \;=\; X(t_1)+V(t_1)\cdot T - \frac{D_M}{2}\cdot T^2, \qquad (1)$$

because the desired and actual positions should agree at the time $t = t_1+T$. T is the deceleration time interval, after which the desired position S_2 should be reached. The simple quadratic dependence reflects the fact that only a constant deceleration D_M is applied. We assume, that $S(t)$ is also moving with a desired velocity V_S, which is supposed to be constant. Thus we can write:

$$S(t_1+T) \;=\; S(t_1)+V_S\cdot T$$

The time interval T is the time needed to arrive at the correct position with desired velocity V_S under full deceleration, and it needs to be determined.

We also have that the velocity has to be the desired velocity V_S, once the correct position has been found:

$$V(t_1+T) \;=\; V_S \;=\; V(t_1) - D_M\cdot T \qquad (2)$$

From this we find the unknown time interval T with:

$$T \;=\; \frac{|V(t_1)-V_S|}{D_M} \qquad (3)$$

The absolute value sign is used here in order to consider also the case when moving to positions with smaller reading than that at the beginning. Implementing this into (1) we get the position, at which we have to start to decelerate at full power:

$$X(t_1) \;=\; S(t_1) - (V(t_1)-V_S)\cdot\frac{|V(t_1)-V_S|}{2\cdot D_M} \qquad (4)$$

The position $X(t)$ as well as the velocity $V(t)$ can be measured during the slew of the telescope. $S(t)$ and V_S are data provided by the observing program, which determines all movements of the telescope. With this, the precise begin of the deceleration t_1 can be found from the permanently ongoing measurements of the position $X(t)$. The velocity $V(t)$ is to be derived as the difference of adjacent position measurements for example (see below).

(For the following we set the desired velocity V_S to zero, since for a satellite like SWAS the sources will not move on sky.) From the time t_1 on the position of the telescope should develop like:

$$\begin{aligned} X(t) \;&=\; X(t_1) + V(t_1)\cdot(t-t_1) - \frac{D_M}{2}\cdot(t-t_1)^2 \\ &=\; S(t_1) - \frac{V(t_1)\cdot|V(t_1)|}{2\cdot D_M} + V(t_1)\cdot(t-t_1) - \frac{D_M}{2}\cdot(t-t_1)^2 \end{aligned} \qquad (5)$$

This is what should be fed into the regulator as a ”reference position” $X_R(t)$ during the deceleration phase. In addition we have for the velocity:

$$V(t) \;=\; V(t_1) - D_M\cdot(t-t_1) \qquad (6)$$

These formulas are correct as long as the instrument moves with the maximum deceleration, which is supposed to be constant. If we use this equation to replace the time interval $t-t_1$, we get from equ.(5):

$$X_R(t) \;=\; S(t) - \frac{V(t)\cdot|V(t)|}{2\cdot D_M} \qquad (7)$$

The index ”*R*” is introduced for the new reference the telescope should follow instead of the $S(t)$ as used before. The basic idea is that instead of feeding instantaneous jumps from one position (S_1) to the next (S_2) into the regulator, one uses now a path, the motors of the telescope drive are able to follow. This equation requires the actual velocity reading from the telescope in order to determine the proper ”reference position” for the servo loop. The trick with this reference is, that the servo loop can start to regulate to this function along the path of deceleration beginning at time t_1, therefore all settling problems can be taken care of during this time. This means, that there is very little precious settling time wasted after having reached the desired position, as it usually happens with most of the position regulators used so far. Equ.(7) serves as a filter for the input data to the regulator, so that the regulator can always follow the input instead of running into a nonlinear regime, where oscillations would start to build up in a disastrous way.

Instead of using this new reference for the period of deceleration only, it is now assumed, that the filter algorithm of equ.(7) is used for any time t. (Therefore we call it X_R, again "*R*" stands for "Reference".) In consequence, the calculated reference is far off the true position as long as the system does not decelerate. Therefore, the calculated reference causes the system to move under maximum acceleration. The deceleration phase on the other hand is supposed to follow the calculated reference position exactly. In order to ensure, that the deceleration can be performed under full regulator control, it is vital that the assumed maximum deceleration D_M is not exactly equal to the maximum value set by the hardware. Instead it has to be slightly lower, so that the path defined by equ.(7) can be followed really under full regulator control.

The problem of data delay

The formulas given before should solve most problems with settling, but, there is always the additional problem of delay between actual position/velocity and the time, when the readings of position/velocity are available. At least the velocity can only be determined after two subsequent readings of the position in most cases. Therefore, the algorithm must be modified taking into account, that there is some time delay present in the system. Let us assume that this time delay "*d*" is always constant. In this case we can replace the velocity *V(t)* by expressions at earlier times, which than takes care of the delay in the system. In fact, to turn it around, we want to calculate the necessary reference position at a certain time in the future with respect to the position readings before, therefore we calculate:

$$V(t) = \frac{5 \cdot X(t-d) - 8 \cdot X(t-2 \cdot d) + 3 \cdot X(t-3 \cdot d)}{2 \cdot d} \qquad (8)$$

The time interval "*d*" is determined by the repetition rate of the position readings of the telescope. This expression is found from the simple expansion:

$$X(t-\delta) = X(t) - V(t) \cdot \delta + \frac{A(t)}{2} \cdot \delta^2$$

Using the values δ = d, 2*d, 3*d, formula (8) is easily derived.

With the same derivation, we also find for the actual position:

$$X(t) = 3 \cdot X(t-d) - 3 \cdot X(t-2 \cdot d) + X(t-3 \cdot d) \qquad (9)$$

Both formulas, equs.(8) and (9), are exact as long as the system moves under constant acceleration or deceleration and are therefore fully justified. This is obviously true for an optimized move of the satellite as described before. Therefore, with these formulas one obtains a very good "prediction" of the present status of the system without being able to measure it at the proper time. If expression (8) is inserted into equ.(7) we have a reasonable estimate of the present velocity *V(t)* from earlier position readings without any negative impact on the servo loop itself.

The algorithm, equ.(7) together with (8) and (9), serves as a filter for the desired position input to the regulator. It guarantees that the required acceleration never exceeds the capacity of the motor drives in use. The (ideal) regulator itself gives maximum output resulting in maximum acceleration anyway, as long as the actual and reference position do not agree. The filter given above therefore does nothing else than the output of an ordinray PI-regulator would provide, if there would be no time constant or bandwidth limitation of the regulator involved. Thus, it is the advantage of the algorithm that it somehow looks into the future, something a normal regulator never could do.

Control of velocity

At least for a telescope, the regulator can control the speed of the motor drives only, and not the position. The control is accomplished by controlling the current fed to the motors. For the reaction wheels on a satellite, the situation may be either the same or the current determines the acceleration instead. In the first case we have in fact a velocity controller instead of a position controller, in the second we have an acceleration controller. For a normal telescope drive we have to consider the first case. On the other hand, the encoders of a telescope usually deliver position data. These must be converted into velocity information, which requires at least two readings of the position or even three as is shown above (equ.(8)). In addition, a proportional/integral regulator requires also the integrated velocity, which is the position (equ.(9)). For both, velocity and position, there must be an algorithm in order to control the reference input to the regulator. The reference position is described by equ.(7). For the reference velocity $V_R(t)$ we can derive an expression in a similar fashion as for the position $X_R(t)$ with:

$$V_R(t) = V_S + Sign[S(t) - X(t)] \cdot \sqrt{2 \cdot D_M \cdot |S(t) - X(t)|} \qquad (10)$$

(This is just the reversed formula of equ.(7) with non-zero V_S.) This calculated reference velocity is now dependent on the actual position $X(t)$, which must be calculated from the earlier readings as is given by equ.(9).

The proportional part of the regulator output is now the difference of the reference velocity $V_R(t)$ as given by equ.(10) (together with equ.(9)) and the true velocity $V(t)$ as calculated from equ.(8)). The integrator part is the difference of the reference position $X_R(t)$ given by equ.(7) (together with equ.(8)) and the true position $X(t)$ as calculated by equ.(9)). The sum of both with some gain factors (to be determined) is the total regulator output. Thus we have for the regulator amplifier output $D_R(t)$:

$$D_R(t) = G_{Prop} \cdot [V_R(t) - V(t)] + G_{Int} \cdot [X_R(t) - X(t)] \qquad (11)$$

G_{Prop} and G_{Int} are the gain factors used for the proportional and integral part of the regulator amplifier. Any eventual differential part in the regulator is not of much interest, because it would force the regulator output into the maximum acceleration mode almost continuously. It does therefore nothing else as is provided already by the proportional/integral part anyway.

Computer simulation

In principle, the algorithm presented here should improve the nodding speed significantly, but, it is certainly necessary to do some kind of testing at least on a computer. For this purpose a program has been developed, which simulates the regulator in all detail. For the telescope a time constant T_S is assumed according to the differential equation:

$$T_S \cdot \frac{\partial V(t)}{\partial t} + V(t) = D_R(t) \qquad (12)$$

$D_R(t)$ is the regulator amplifier output as given by equ.(11). This simulates the time delay of the response of the motors. $V_R(t)$ is calculated according to the algorithm as described above. For the movement of the telescope the solution of this differential equation is integrated, as long as the hardware limits A_M and V_M are not reached. The nod time of t_{Nod} = 2.89 sec for a 1 degree nod is defined as the time, where the desired value of 1 degree is reached the first time (see Fig.2). It is practically identical with the value found using equ.(3), which gives t_{Nod} = 2.91 sec using A_M = 0.5°/sec² and D_M = o.45°/sec². (If a theoretical deceleration of 0.5°/sec² is used, one finds a nod time of 2.82 sec.) It is obvious, that there is almost zero time wasted, although the maximum deceleration used is 0.45°/sec² only. Fig.2 shows the difference between actual and planned position. By the way, in case of longer delay of position readings, as is normal for satellite applications for example, the nod time is slightly prolonged, because the deceleration phase cannot be controlled that precisely as it is obviously necessary. The longer the delay, the less reliable the position and velocity predictions become.

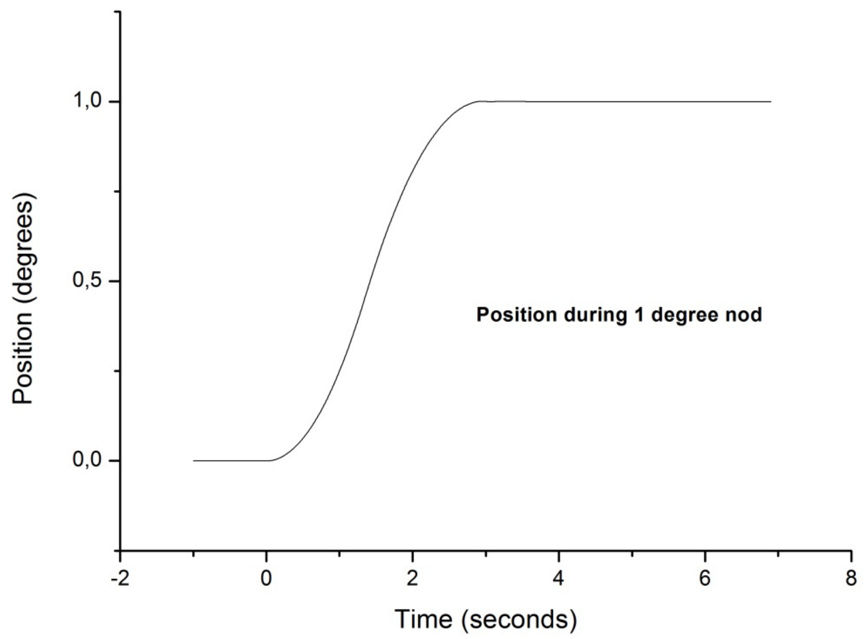

Fig.1: *Simulation of a 1 Degree nod. The assumed acceleration A_M was $0.5°/sec^2$ and the maximum deceleration was $0.45°/sec^2$. The nod movement starts at t=0.*

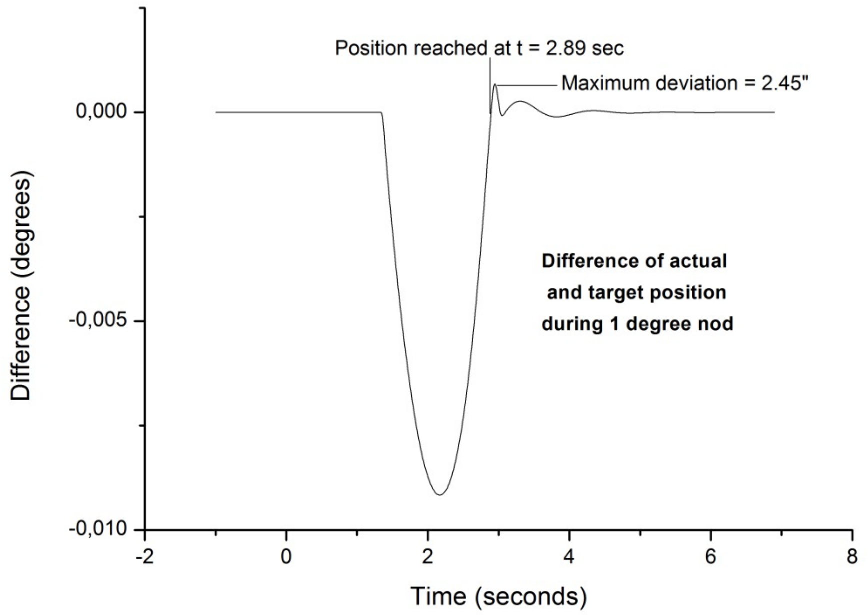

Fig.2: *Simulation of a 1 Degree nod. Shown is the difference between target and actual position. During deceleration the error is relatively large at the beginning, but the final position is found with high accuracy.*

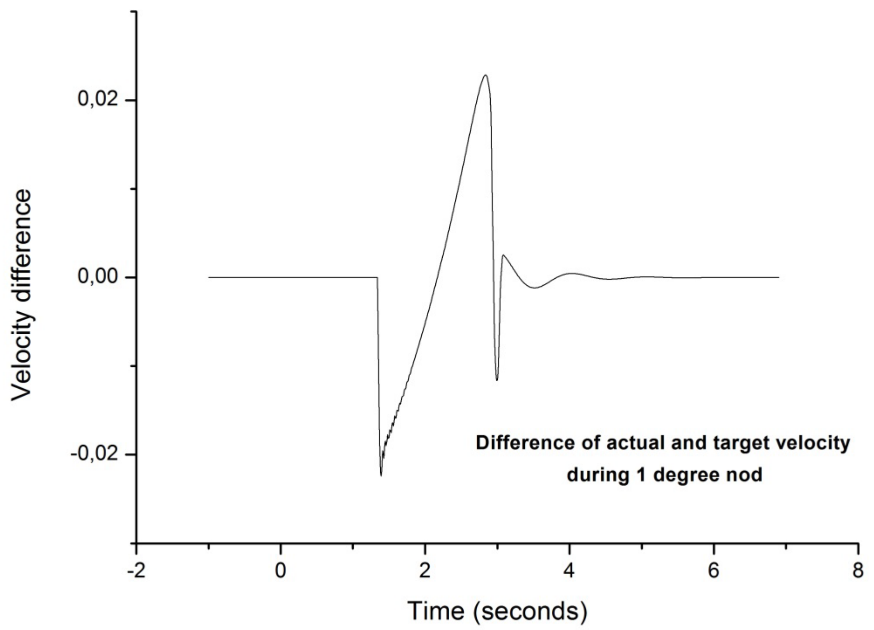

Fig.3: *Difference of actual and target velocity during 1 degree nod*

Gain control

During the simulations as well as on Gornergrat, a significant “trembling” of the system can occur even after having reached the final position. This is caused due to the fact, that the assumed acceleration or deceleration is not valid with normal settings of the proportional gain. In principle such oscillations would not hurt, but the motors are running under constant maximum current which switches rapidly in direction. This is certainly not good for the gears involved, and is not acceptable for a satellite system because of the electrical power constraints. Therefore, it was investigated, how these oscillations can be stopped. A very simple solution was found, by changing the regulator gain when having reached the desired position. In that instant, where the sign of the velocity changes during the nod, it is clear that the position has been found and an almost negligible time period is needed only for final settling. In the simulation software the gain of the proportional amplifier as derived from equ.(10) is set to zero at the instant the velocity changes sign, while the gain of the integrator remains unchanged. It is set back to the normal value in case the integrator output exceeds a rather small, but preselected value. This eliminates the unfavorable trembling. Some trembling is seen in Fig.3 at the beginning of the deceleration phase of the nod. With time it diminishes once the movement is coming closer to the final position. After turn-off of the proportional amplifier it disappears completely. The rather slow oscillation seen after this is due to the system time constant of 0.3 sec assumed for the calculation. It is rather remarkable that the overshoot above the 1 degrees position is less than 3 arcseconds only (see Fig.2). This is only a very small fraction of a typical beamwidth of a radio telescope and therefore insignificant.

Usually, when simulating regulator systems, it is essential that the impact of noise is also investigated. For this some noise based on random numbers was inserted into the servo loop and the possible changes on the regulator performance were watched. As it turned out, the regulator system is not very sensitive to noise, but the deceleration must be chosen slightly lower than without noise. This is understandable because the regulator must try to compensate for the noise fluctuations while decelerating near the limits set to the hardware. In order to do this some more headroom is needed for the deceleration in use.

Conclusions

By means of computer simulation it is established, that a servo loop can be operated for a telescope, which guarantees a minimum nod time under all circumstances. For this four measures are needed:

1.) The velocity must be controlled with a "sawtooth" input. All input data to the regulator must be modified according to the algorithms given by equs.(7) and (10).

2.) The deceleration must be done with a slightly lower value, than the hardware would allow at maximum. This moves the settling time interval into the deceleration period. In consequence there is almost no additional settling time needed afterwards.

3.) Less frequent position readings require, that the true position and velocity are extrapolated to the actual time by using equs.(9) and (8). This overcomes the difficulty of time delay in the system. The total bandwidth of the servo loop can therefore be made much higher than without this method.

4.) The gain of the regulator amplifier must be controlled in a way, which avoids "steady state" oscillations. This can be realized by turning off the proportional gain once the system has reached the desired position.

All four measures together show excellent performance in a computer simulation. It is found, that under assumptions as they might be typical for a satellite like SWAS, the actual nod time is not more than about 0.2 seconds longer than the minimum theoretical time needed. It is almost independent on the actual parameters like time constant or time delay, and, most remarkably, even on nod amplitude! This is also true, in case condition 4.) is not fulfilled.

As a recommendation I would like to say, that all measures should be taken to improve the nodding speed of SWAS, because the nodding speed determines the efficiency of the use of the precious observing time. This is not only the rational for the application of the filter algorithm presented here, but it is also a strong argument to make the time period of position readings on SWAS as short as possible. Another problem might be to decide, what is really controlled by the regulator. For the Gornergrat telescope it is the speed of the motors which is in fact controlled by the servo loop. For SWAS it might be the acceleration, because the current set by the controller determines the change of speed and not the speed of the wheels itself (at least I believe so). In any case it is not the position, the system can regulate directly, thus the servo loop must be designed accordingly.

Note added for groundbased telescopes

Usually, there is an additional problem showing up, which is typical for all such regulators, that is a drag error arising from the requirement to follow a source at some speed. Normally it is almost impossible to overcome this drag problem, but a modification of equ.(11) has been found, which might be useful. If one modifies equ.(11) to

$$D_R(t) = G_{Vel} \cdot V_S + G_{Prop} \cdot [V_R(t) - V(t)] + G_{Int} \cdot [X_R(t) - X(t)] \quad , \qquad (11)$$

an additional offset is generated, which causes the two terms in the brackets to become zero in equilibrium. This means, that the additional term V_S takes care of the needed offset for proper velocity setting. Tests with the software have shown, that the drag offset can be made nearly zero. But, for this it must be recognized, that the required gain-factor G_{Vel} as input to the motors must be well known for obtaining exactly the speed V_S, and should be determined by experiment. With aging hardware it is also very likely that the response changes, thus the calibration has to be repeated from time to time. It should also be noted, that the setting of the threshold for turning on or off the proportional amplifier gain must be found by very careful investigation.

Additional note for use in accelerated systems

In case the desired position includes also an acceleration term like

$$S(t) = S(0) + V_S \cdot t + \frac{A_S}{2} \cdot t^2, \text{ and}$$

$$V_S = V(0) + A_S \cdot t,$$

then equs.(7) and (10) must be modified:

$$X_R(t) = S(t) + \frac{[V_S(t) - V(t)]^2}{2 \cdot [V_{ZV} \cdot D_M - A_S]} \qquad (7)$$

with $V_{ZV} = Sign[V_S(t) - V(t)]$ and

$$V_R(t) = V_S(t) + V_{ZX} \cdot \sqrt{2 \cdot |[V_{ZX} \cdot D_M + A_S] \cdot [S(t) - X(t)]|} \qquad (10)$$

with $V_{ZX} = Sign[S(t) - V(t)]$

Equ.(11) remains the same as before:

$$D_R = G_{Vel} \cdot V_S(t) + G_{Prop} \cdot [V_R(t) - V(t)] + G_{Int} \cdot [X_R(t) - X(t)] \quad (11)$$

This might be useful when following the course of satellites or other objects, which are not moving with constant angular speed.